

Irreducible Hamiltonian BRST symmetry for reducible first-class systems

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Abstract

An irreducible Hamiltonian BRST quantization method for reducible first-class systems is proposed. The general theory is illustrated on a two-stage reducible model, the link with the standard reducible BRST treatment being also emphasized.

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1 Introduction

It is well known that the Hamiltonian BRST formalism [1]–[6] stands for one of the strongest and most popular quantization methods for theories with first-class constraints. This method can be applied to irreducible, as well as reducible first-class systems. In the irreducible case the ghosts can be regarded as one-forms dual to the vector fields associated with the first-class constraints. This interpretation fails in the reducible framework, being necessary to introduce ghosts for ghosts together with their canonical conjugated momenta (antighosts). The ghosts for ghosts ensure the incorporation of the

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reducibility relations within the cohomology of the exterior derivative along the gauge orbits, while their canonical momenta are required in order to kill the higher-resolution-degree nontrivial co-cycles from the homology of the Koszul-Tate differential. Similar considerations apply to the antifield BRST treatment [6]–[11].

Another way of approaching reducible systems is based on the idea of replacing such a system with an irreducible one [6], [12]. This idea has been enforced in the framework of the irreducible quantization of reducible theories from the point of view of the antifield-BRST formulation [13], as well as of the antifield BRST-anti-BRST method [14]. Some applications of these treatments can be found in [15]–[16]. At the level of the Hamiltonian symmetry, this type of procedure has been developed so far only in the case of some peculiar models [17]–[18], but a general theory covering on-shell reducible first-class systems has not yet been given.

In view of this, here we propose an irreducible Hamiltonian BRST procedure for quantizing on-shell reducible first-class theories. In this light, we enforce the following steps: (i) we transform the original reducible first-class constraints into some irreducible ones on a larger phase-space in a manner that allows the substitution of the BRST quantization of the reducible system by that of the irreducible theory; (ii) we quantize the irreducible system accordingly the Hamiltonian BRST formalism. As a consequence, the ghosts for ghosts and their antighosts do not appear in our formalism.

The paper is organized in five sections. Section 2 deals with the derivation of an irreducible set of first-class constraints associated with the original reducible one by means of constructing an irreducible Koszul-Tate complex. The irreducible Koszul-Tate complex is obtained by requiring that all the antighost number one co-cycles from the reducible case become trivial under an appropriate redefinition of the antighost number one antighosts. This procedure leads to the introduction of new canonical variables and antighosts. In section 3 we infer the irreducible BRST symmetry corresponding to the irreducible constraint set deduced in section 2 and prove that we can replace the Hamiltonian BRST quantization of the reducible system with that of the irreducible theory. Section 4 illustrates our method in the case of a two-stage reducible model involving three-form gauge fields. In section 5 we present the main conclusions of the paper.

2 Derivation of an irreducible first-class constraint set. Irreducible Koszul-Tate differential

2.1 Setting the problem

Our starting point is a Hamiltonian system with the phase-space locally described by N canonical pairs $z^A = (q^i, p_i)$, subject to the first-class constraints

$$G_{a_0}(q^i, p_i) \approx 0, \quad a_0 = 1, \dots, M_0, \quad (1)$$

which are assumed to be on-shell L -stage reducible. We suppose that there are no second-class constraints in the theory (if any, they can be eliminated with the help of the Dirac bracket). The first-class property of the constraints (1) is expressed by

$$[G_{a_0}, G_{b_0}] = C_{a_0 b_0}^{c_0} G_{c_0}, \quad (2)$$

while the reducibility relations are written as

$$Z_{a_1}^{a_0} G_{a_0} = 0, \quad a_1 = 1, \dots, M_1, \quad (3)$$

$$Z_{a_2}^{a_1} Z_{a_1}^{a_0} = C_{a_2}^{a_0 b_0} G_{b_0}, \quad a_2 = 1, \dots, M_2, \quad (4)$$

\vdots

$$Z_{a_L}^{a_{L-1}} Z_{a_{L-1}}^{a_{L-2}} = C_{a_L}^{a_{L-2} b_0} G_{b_0}, \quad a_L = 1, \dots, M_L, \quad (5)$$

with the symbol $[,]$ denoting the Poisson bracket. For definiteness we approach here the bosonic case, but our analysis can be easily extended to fermions modulo introducing some appropriate sign factors. The first-order structure functions $C_{a_0 b_0}^{c_0}$ may involve the phase-space coordinates (open gauge algebra) and are antisymmetric in the lower indices. The reducibility functions $(Z_{a_{k+1}}^{a_k})_{k=0, \dots, L-1}$ and the coefficients $(C_{a_{k+2}}^{a_k b_0})_{k=0, \dots, L-2}$ appearing in the right hand-side of the relations (4–5) may also depend on the canonical variables, and, in addition, $C_{a_2}^{a_0 b_0}$ should be antisymmetric in the upper indices.

The standard Hamiltonian BRST symmetry for the above on-shell reducible first-class Hamiltonian system, $s_R = \delta_R + D_R + \dots$, contains two crucial operators. The Koszul-Tate differential δ_R realizes an homological

resolution of smooth functions defined on the first-class constraint surface (1), while the model of longitudinal exterior derivative along the gauge orbits D_R is a differential modulo δ_R and accounts for the gauge invariances. The degree of δ_R is called antighost number (*antigh*), the degree of D_R is named pure ghost number (*pg*), while the overall degree of the BRST differential is called ghost number (*gh*) and is defined like the difference between the pure ghost number and the antighost number ($\text{antigh}(\delta_R) = -1$, $\text{pg}(D_R) = 1$, $\text{gh}(s_R) = 1$). In the case of a first-stage reducible Hamiltonian system ($L = 1$) the proper construction of δ_R relies on the introduction of the generators (antighosts) \mathcal{P}_{a_0} and P_{a_1} , with the Grassmann parity (ϵ) and the antighost number given by

$$\epsilon(\mathcal{P}_{a_0}) = 1, \epsilon(P_{a_1}) = 0, \text{antigh}(\mathcal{P}_{a_0}) = 1, \text{antigh}(P_{a_1}) = 2, \quad (6)$$

on which δ_R is set to act like

$$\delta_R \mathcal{P}_{a_0} = -G_{a_0}, \quad (7)$$

$$\delta_R P_{a_1} = -Z_{a_1}^{a_0} \mathcal{P}_{a_0}, \quad (8)$$

being understood that

$$\delta_R z^A = 0. \quad (9)$$

The antighosts P_{a_1} are required in order to “kill” the antighost number one nontrivial co-cycles

$$\mu_{a_1} = Z_{a_1}^{a_0} \mathcal{P}_{a_0}, \quad (10)$$

(which are due to the definitions (7) and the reducibility relations (3)) in the homology of δ_R , and thus restore the acyclicity of the Koszul-Tate differential at nonvanishing antighost numbers. For a two-stage reducible Hamiltonian system ($L = 2$), one should supplement the antighost spectrum from the first-stage case with the antighosts P_{a_2} , displaying the characteristics $\epsilon(P_{a_2}) = 1$, $\text{antigh}(P_{a_2}) = 3$, and define the action of δ_R on them through $\delta_R P_{a_2} = -Z_{a_2}^{a_1} P_{a_1} - \frac{1}{2} C_{a_2}^{a_0 b_0} \mathcal{P}_{b_0} \mathcal{P}_{a_0}$. The acyclicity of δ_R is thus achieved by making exact the antighost number two nontrivial co-cycles $\mu_{a_2} = Z_{a_2}^{a_1} P_{a_1} + \frac{1}{2} C_{a_2}^{a_0 b_0} \mathcal{P}_{b_0} \mathcal{P}_{a_0}$, which are present on behalf of the definitions (8) and the reducibility relations (4). The process goes along the same lines at higher antighost numbers. In the general situation of an L -stage reducible Hamiltonian theory, the antighost spectrum contains the variables

\mathcal{P}_{a_0} and $(P_{a_k})_{k=1,\dots,L}$, with $\epsilon(P_{a_k}) = k + 1 \bmod 2$, $\text{antigh}(P_{a_k}) = k + 1$, the actions of δ_R on higher antighost number antighosts being taken in such a way to ensure the acyclicity of the Koszul-Tate differential at nonvanishing antighost numbers.

The problem to be investigated in the sequel is the derivation of an irreducible set of first-class constraints associated with the L -stage reducible one (1). Our basic idea is to redefine the antighosts \mathcal{P}_{a_0} in such a way that the new co-cycles of the type (10) are trivial. Then, the antighosts P_{a_1} ensuring the triviality of the co-cycles (10) are no longer necessary, so they will be discarded from the Koszul-Tate complex. Moreover, the absence of these antighosts implies the absence of nontrivial co-cycles at antighost number greater than one, hence the antighosts $(P_{a_k})_{k=2,\dots,L}$ will also be discarded from the antighost spectrum. The enforcement of this idea leads to an irreducible set of first-class constraints underlying some physical observables that coincide with those deriving from the original reducible system. In order to simplify the presentation we initially approach the cases $L = 1, 2$, and further generalize the results to an arbitrary L .

2.2 The case $L = 1$

We begin with the definitions (7–8) and the reducibility relations (3). As we have previously mentioned, we redefine the antighosts \mathcal{P}_{a_0} like

$$\mathcal{P}_{a_0} \rightarrow \mathcal{P}'_{a_0} = \mathcal{P}_{a_0} - Z^{b_0}_{b_1} \bar{D}^{b_1}_{c_1} A^{c_1}_{a_0} \mathcal{P}_{b_0}, \quad (11)$$

where $\bar{D}^{b_1}_{c_1}$ stands for the inverse of $D^{b_1}_{c_1} = Z^{c_0}_{c_1} A^{b_1}_{c_0}$, while $A^{c_1}_{a_0}$ are some functions that may involve at most the variables z^A and are chosen to satisfy $\text{rank}(D^{b_1}_{c_1}) = M_1$. Next, we replace (7) with

$$\delta \mathcal{P}'_{a_0} = -G_{a_0}. \quad (12)$$

The definitions (12) imply some co-cycles of the type (10)

$$\mu'_{a_1} = Z^{a_0}_{a_1} \mathcal{P}'_{a_0}, \quad (13)$$

which are found trivial on behalf of (11), namely, $\mu'_{a_1} \equiv 0$. Thus, the definitions (12) do not lead to nontrivial co-cycles at antighost number one, therefore the antighosts P_{a_1} will be discarded. Moreover, formulas (12) are helpful

at deriving some irreducible first-class constraints corresponding to (1). For this reason in (12) we used the notation δ instead of δ_R . The derivation of the irreducible first-class constraints relies on enlarging the phase-space with the bosonic canonical pairs (y^{a_1}, π_{a_1}) , where the momenta π_{a_1} are demanded to be nonvanishing solutions to the equations

$$D^{b_1}_{a_1} \pi_{b_1} = \delta \left(-Z^{a_0}_{a_1} \mathcal{P}_{a_0} \right). \quad (14)$$

As $D^{b_1}_{a_1}$ is invertible, the nonvanishing solutions to (14) implement the irreducibility. This is because the equations (14) possess nonvanishing solutions if and only if

$$\delta \left(Z^{a_0}_{a_1} \mathcal{P}_{a_0} \right) \neq 0, \quad (15)$$

hence if and only if (10) are not co-cycles. Inserting (14) in (12) and using (11), we arrive at

$$\delta \mathcal{P}_{a_0} = -G_{a_0} - A^{a_1}_{a_0} \pi_{a_1}, \quad (16)$$

which emphasize the irreducible constraints

$$\gamma_{a_0} \equiv G_{a_0} + A^{a_1}_{a_0} \pi_{a_1} \approx 0. \quad (17)$$

From (17) it is easy to see that

$$\pi_{a_1} = \bar{D}^{b_1}_{a_1} Z^{b_0}_{b_1} \gamma_{b_0}, \quad G_{a_0} = \left(\delta^{b_0}_{a_0} - Z^{b_0}_{b_1} \bar{D}^{b_1}_{a_1} A^{a_1}_{a_0} \right) \gamma_{b_0}, \quad (18)$$

so

$$[\gamma_{a_0}, \gamma_{b_0}] = \bar{C}^{c_0}_{a_0 b_0} \gamma_{c_0}, \quad (19)$$

for some $\bar{C}^{c_0}_{a_0 b_0}$. Thus, the irreducible constraints (17) are first-class. In the meantime, if we take the standard action of δ on the phase-space coordinates

$$\delta z^A = 0, \quad \delta z^{A_1} = 0, \quad (20)$$

with $z^{A_1} = (y^{a_1}, \pi_{a_1})$, then formulas (16) and (20) completely define an irreducible Koszul-Tate complex corresponding to an irreducible system based on the first-class constraints (17).

At this point we underline two important observations. First, the number of physical degrees of freedom of the irreducible system coincides with the original one. Indeed, in the reducible case there are N canonical pairs and $M_0 - M_1$ independent first-class constraints, hence $N - M_0 + M_1$ physical

degrees of freedom. In the irreducible situation there are $N + M_1$ canonical pairs and M_0 independent first-class constraints, therefore as many physical degrees of freedom as in the reducible case. Second, from (14) it results (due to the invertibility of $D_{a_1}^{b_1}$) that the momenta π_{a_1} are δ -exact. These results represent two desirable features, which will be requested also in connection with higher-order reducible Hamiltonian systems. Anticipating a bit, we notice that for higher-order reducible theories it is necessary to further add some supplementary canonical variables and antighost number one antighosts. While the former request indicates the number of new canonical pairs and first-class constraints to be added within the irreducible framework, the latter ensures, as it will be further seen, that there exists a proper redefinition of the antighost number one antighosts that makes trivial the co-cycles from the reducible approach.

2.3 The case $L = 2$

Now, we start from the reducibility relations (3–4) and intend to preserve the definitions (16) and (20) with respect to the irreducible Koszul-Tate differential δ . Nevertheless, there appear two difficulties. First, the matrix $D_{c_1}^{b_1}$ is not invertible now due to the second-stage reducibility relations (4). Rather, it possesses $Z_{a_2}^{c_1}$ as on-shell null vectors

$$D_{c_1}^{b_1} Z_{a_2}^{c_1} = A_{c_0}^{b_1} Z_{c_1}^{c_0} Z_{a_2}^{c_1} = C_{a_2}^{c_0 d_0} A_{c_0}^{b_1} G_{d_0} \approx 0, \quad (21)$$

hence the transformations (11) fail to be correct. We will still maintain the definitions (16) and will subsequently show that there exists a redefinition of the antighosts \mathcal{P}_{a_0} that brings the constraint functions γ_{a_0} under the form (17). Second, the irreducible first-class constraints (17) are not enough in order to maintain the initial number of physical degrees of freedom in connection with the irreducible theory, so they should be supplemented with M_2 new constraints in such a way to ensure a first-class irreducible behaviour of the overall constraint set. Thus, we must introduce some fermionic antighost number one antighosts \mathcal{P}_{a_2} and set

$$\delta \mathcal{P}_{a_2} = -\gamma_{a_2}, \quad (22)$$

where $\gamma_{a_2} \approx 0$ denote the new first-class constraints. On the one hand, we should infer the concrete form of γ_{a_2} such that the new Koszul-Tate complex

based on the definitions (16), (20) and (22) is irreducible, and, on the other hand, we should prove that it is possible to perform a redefinition of the antighosts \mathcal{P}_{a_0} (eventually involving also the new antighosts) such that (16) is gained. This goes as follows. We begin with the relations (16) on which we apply $Z_{b_1}^{a_0}$, and obtain

$$\delta \left(Z_{b_1}^{a_0} \mathcal{P}_{a_0} \right) = -D_{b_1}^{a_1} \pi_{a_1}. \quad (23)$$

Formulas (21) make permissible a representation of $D_{b_1}^{a_1}$ under the form

$$D_{b_1}^{a_1} = \delta_{b_1}^{a_1} - Z_{a_2}^{a_1} \bar{D}_{b_2}^{a_2} A_{b_1}^{b_2} + A_{a_0}^{a_1} C_{c_2}^{a_0 b_0} \bar{D}_{b_2}^{c_2} A_{b_1}^{b_2} G_{b_0}, \quad (24)$$

where $\bar{D}_{b_2}^{a_2}$ is the inverse of $D_{b_2}^{a_2} = Z_{b_2}^{a_1} A_{a_1}^{a_2}$ and $A_{a_1}^{a_2}$ are some functions that may depend at most on z^A and are taken to fulfill $\text{rank} \left(D_{b_2}^{a_2} \right) = M_2$. Inserting (24) in (23), we infer that

$$\delta \left(Z_{b_1}^{a_0} \mathcal{P}_{a_0} \right) = -\pi_{b_1} + Z_{a_2}^{a_1} \bar{D}_{b_2}^{a_2} A_{b_1}^{b_2} \pi_{a_1} - C_{c_2}^{a_0 b_0} \bar{D}_{b_2}^{c_2} A_{b_1}^{b_2} A_{a_0}^{a_1} \pi_{a_1} G_{b_0}, \quad (25)$$

which turns into

$$\delta \left(Z_{b_1}^{a_0} \mathcal{P}_{a_0} - C_{c_2}^{a_0 b_0} \bar{D}_{b_2}^{c_2} A_{b_1}^{b_2} G_{b_0} \mathcal{P}_{a_0} \right) = -\pi_{b_1} + \bar{D}_{b_2}^{a_2} A_{b_1}^{b_2} Z_{a_2}^{a_1} \pi_{a_1}, \quad (26)$$

on account on the one hand of the antisymmetry of $C_{c_2}^{a_0 b_0}$, which implies the relations $C_{c_2}^{a_0 b_0} A_{a_0}^{a_1} \pi_{a_1} G_{b_0} = C_{c_2}^{a_0 b_0} \gamma_{a_0} G_{b_0}$, and, on the other hand, of the definitions (16) and (20). The second term in the right hand-side of (26) implies that π_{a_1} are not δ -exact. An elegant manner of complying with the requirement on the δ -exactness of these momenta is to take the functions γ_{a_2} like

$$\gamma_{a_2} \equiv Z_{a_2}^{a_1} \pi_{a_1}, \quad (27)$$

so $\delta \mathcal{P}_{a_2} = -Z_{a_2}^{a_1} \pi_{a_1}$, which then leads, via (26), to

$$\pi_{b_1} = \delta \left(-Z_{b_1}^{a_0} \mathcal{P}_{a_0} + C_{c_2}^{a_0 b_0} \bar{D}_{b_2}^{c_2} A_{b_1}^{b_2} G_{b_0} \mathcal{P}_{a_0} - \bar{D}_{b_2}^{a_2} A_{b_1}^{b_2} \mathcal{P}_{a_2} \right). \quad (28)$$

On behalf of (27) and of the fact that the functions $Z_{a_2}^{a_1}$ have no null vectors, we find that (22) imply no nontrivial co-cycles. In this way the first task is achieved. Introducing (28) in (16), we arrive at

$$\delta \mathcal{P}_{a_0}'' = -G_{a_0}, \quad (29)$$

where

$$\mathcal{P}_{a_0}'' = \mathcal{P}_{a_0} - Z_{b_1}^{b_0} A_{a_0}^{b_1} \mathcal{P}_{b_0} + C_{c_2}^{c_0 b_0} \bar{D}_{b_2}^{c_2} A_{b_1}^{b_2} A_{a_0}^{b_1} G_{b_0} \mathcal{P}_{c_0} - \bar{D}_{b_2}^{a_2} A_{b_1}^{b_2} A_{a_0}^{b_1} \mathcal{P}_{a_2}. \quad (30)$$

Formula (30) expresses a redefinition of the antighost number one antighosts that is in agreement with (16). It is precisely the requirement on the δ -exactness of the momenta π_{a_1} that allows us to deduce (30). Thus, the second task is also attained.

It remains to be proved that (29) also gives no nontrivial co-cycles. From (29) we obtain the co-cycles

$$\mu_{a_1}'' = Z_{a_1}^{a_0} \mathcal{P}_{a_0}'', \quad (31)$$

at antighost number one. After some computation, we find that they are trivial as

$$\mu_{a_1}'' = \delta \left(-\frac{1}{2} C_{c_2}^{a_0 b_0} \bar{D}_{b_2}^{c_2} A_{a_1}^{b_2} \mathcal{P}_{a_0}'' \mathcal{P}_{b_0}'' \right). \quad (32)$$

In conclusion, we associated an irreducible Koszul-Tate complex based on the definitions (16), (20) and (22) with the starting second-stage reducible one. This complex underlies the irreducible constraint functions (17) and (27), which can be shown to be first-class. Indeed, with the help of (17) and (27) we get that

$$\pi_{b_1} = \left(Z_{b_1}^{a_0} - C_{c_2}^{a_0 b_0} \bar{D}_{b_2}^{c_2} A_{b_1}^{b_2} G_{b_0} \right) \gamma_{a_0} + \bar{D}_{b_2}^{a_2} A_{b_1}^{b_2} \gamma_{a_2}, \quad (33)$$

$$G_{a_0} = \left(\delta_{a_0}^{c_0} - Z_{b_1}^{c_0} A_{a_0}^{b_1} + C_{c_2}^{c_0 b_0} \bar{D}_{b_2}^{c_2} A_{b_1}^{b_2} A_{a_0}^{b_1} G_{b_0} \right) \gamma_{c_0} - \bar{D}_{b_2}^{a_2} A_{b_1}^{b_2} A_{a_0}^{b_1} \gamma_{a_2}. \quad (34)$$

Evaluating the Poisson brackets among the constraint functions $(\gamma_{a_0}, \gamma_{a_2})$ with the help of (33–34), we find that they weakly vanish on the surface $\gamma_{a_0} \approx 0, \gamma_{a_2} \approx 0$, hence (17) and (27) are first-class. This completes the case $L = 2$.

2.4 Generalization to an arbitrary L

Now, we are in the position to generalize the above discussion to an arbitrary L . Acting like in the previous cases, we enlarge the phase-space with the bosonic canonical pairs $z^{A_{2k+1}} = \left(y^{a_{2k+1}}, \pi_{a_{2k+1}} \right)_{k=0, \dots, a}$ and construct an irreducible Koszul-Tate complex based on the definitions

$$\delta z^A = 0, \quad \delta z^{A_{2k+1}} = 0, \quad k = 0, \dots, a, \quad (35)$$

$$\delta\mathcal{P}_{a_0} = -\gamma_{a_0}, \quad (36)$$

$$\delta\mathcal{P}_{a_{2k}} = -\gamma_{a_{2k}}, \quad k = 1, \dots, b, \quad (37)$$

which emphasize the irreducible constraints

$$\gamma_{a_0} \equiv G_{a_0} + A_{a_0}^{a_1} \pi_{a_1} \approx 0, \quad (38)$$

$$\gamma_{a_{2k}} \equiv Z_{a_{2k}}^{a_{2k-1}} \pi_{a_{2k-1}} + A_{a_{2k}}^{a_{2k+1}} \pi_{a_{2k+1}} \approx 0, \quad k = 1, \dots, b. \quad (39)$$

The antighosts $(\mathcal{P}_{a_{2k}})_{k=0, \dots, b}$ are all fermionic with antighost number one, while the notations a and b signify

$$a = \begin{cases} \frac{L-1}{2}, & \text{for } L \text{ odd,} \\ \frac{L}{2} - 1, & \text{for } L \text{ even,} \end{cases} \quad b = \begin{cases} \frac{L-1}{2}, & \text{for } L \text{ odd,} \\ \frac{L}{2}, & \text{for } L \text{ even.} \end{cases} \quad (40)$$

In order to avoid confusion, we use the conventions $f^{a_k} = 0$ if $k < 0$ or $k > L$. The matrices of the type $A_{a_k}^{a_{k+1}}$ implied in (38–39) may involve at most the original variables z^A and are taken to fulfill the relations

$$\text{rank} \left(D_{b_k}^{a_k} \right) \approx \sum_{i=k}^L (-)^{k+i} M_i, \quad k = 1, \dots, L-1, \quad (41)$$

$$\text{rank} \left(D_{b_L}^{a_L} \right) = M_L, \quad (42)$$

where $D_{b_k}^{a_k} = Z_{b_k}^{a_{k-1}} A_{a_{k-1}}^{a_k}$. We remark that the choice of the functions $A_{a_{k-1}}^{a_k}$ is not unique. Moreover, for a definite choice of these functions, the equations (41–42) are unaffected if we modify $A_{a_{k-1}}^{a_k}$ as

$$A_{a_{k-1}}^{a_k} \rightarrow A_{a_{k-1}}^{a_k} + \mu_{b_{k-2}}^{a_k} Z_{a_{k-1}}^{b_{k-2}}, \quad (43)$$

hence these functions carry some ambiguities.

Acting like in the cases $L = 1, 2$, after some computation we find the relations

$$\pi_{a_{2k+1}} = m_{a_{2k+1}}^{a_{2j}} \gamma_{a_{2j}}, \quad k = 0, \dots, a, \quad (44)$$

$$G_{a_0} = m_{a_0}^{a_{2j}} \gamma_{a_{2j}}, \quad (45)$$

for some functions $m_{a_{2k+1}}^{a_{2j}}$ and $m_{a_0}^{a_{2j}}$. Computing the Poisson brackets among the constraint functions in (38–39), we find that they weakly vanish on the surface (38–39), hence they form a first-class set. This ends the general construction of an irreducible Koszul-Tate complex associated with the original on-shell L -stage reducible Hamiltonian system.

3 Irreducible BRST symmetry for on-shell reducible Hamiltonian systems

3.1 Derivation of the irreducible BRST symmetry

The first step in deriving an irreducible BRST symmetry for the investigated on-shell L -stage reducible Hamiltonian system has been implemented by constructing an irreducible Koszul-Tate complex based on the irreducible first-class constraints (38–39). The BRST symmetry corresponding to this irreducible first-class constraint set can be decomposed like

$$s = \delta + D + \dots, \quad (46)$$

where δ is the irreducible Koszul-Tate differential generated in the previous section, D represents the longitudinal exterior derivative along the gauge orbits, and the other pieces, generically denoted by “ \dots ”, ensure the nilpotency of s . The ghost spectrum of the longitudinal exterior complex includes only the fermionic ghosts $\eta^\Delta \equiv (\eta^{a_{2k}})_{k=0,\dots,b}$ with pure ghost number one ($pg\hbar(\eta^{a_{2k}}) = 1$), respectively associated with the irreducible first-class constraints (38–39), to be redenoted by $\gamma_\Delta \equiv (\gamma_{a_{2k}})_{k=0,\dots,b}$. The standard definitions of D acting on the generators from the longitudinal exterior complex read as

$$DF = [F, \gamma_\Delta] \eta^\Delta, \quad (47)$$

$$D\eta^\Delta = \frac{1}{2} C_{\Delta'\Delta''}^\Delta \eta^{\Delta'} \eta^{\Delta''}, \quad (48)$$

where F can be any function of the variables z^A , $(z^{A_{2k+1}})_{k=0,\dots,a}$, and $C_{\Delta'\Delta''}^\Delta$ stand for the first-order structure functions corresponding to the first-class constraint functions

$$[\gamma_\Delta, \gamma_{\Delta'}] = C_{\Delta\Delta'}^{\Delta''} \gamma_{\Delta''}. \quad (49)$$

The action of δ can be extended to the ghosts through

$$\delta\eta^\Delta = 0, \quad (50)$$

with $\text{antigh}(\eta^\Delta) = 0$, such that both the nilpotency and acyclicity of the irreducible Koszul-Tate differential are maintained, while D can be appropriately extended to the antighosts in such a way to become a differential

modulo δ on account of the first-class behaviour of the irreducible constraints. On these grounds, the homological perturbation theory [19]–[22] guarantees the existence of a nilpotent BRST symmetry s of the form (46) associated with the irreducible first-class constraints (38–39).

3.2 Link with the standard reducible BRST symmetry

In the sequel we establish the correlation between the standard Hamiltonian BRST symmetry corresponding to the starting reducible first-class system and that associated with the irreducible one, investigated in the above subsection. In this light we show that the physical observables of the two theories coincide. Let F be an observable of the irreducible system. Consequently, it satisfies the equations

$$[F, \gamma_{a_{2k}}] \approx 0, \quad k = 0, \dots, b, \quad (51)$$

where the weak equality refers to the surface (38–39). Using the relations (44–45), we then find that F also fulfills the equations

$$[F, G_{a_0}] = [F, m_{a_0}^{a_{2j}}] \gamma_{a_{2j}} + [F, \gamma_{a_{2j}}] m_{a_0}^{a_{2j}} \approx 0, \quad (52)$$

$$[F, \pi_{a_{2k+1}}] = [F, m_{a_{2k+1}}^{a_{2j}}] \gamma_{a_{2j}} + [F, \gamma_{a_{2j}}] m_{a_{2k+1}}^{a_{2j}} \approx 0, \quad k = 0, \dots, a, \quad (53)$$

on this surface. So, every observable of the irreducible theory should verify the equations (52–53) on the surface (38–39). Now, we observe that the first-class surface described by the relations (38–39) is equivalent with that described by the equations

$$G_{a_0} \approx 0, \quad \pi_{a_{2k+1}} \approx 0, \quad k = 0, \dots, a. \quad (54)$$

Indeed, it is clear that when (54) hold, (38–39) hold, too. The converse results from (44–45), which show that if (38–39) hold, then (54) also hold. This proves the equivalence between the first-class surfaces (38–39) and (54). As a consequence, we have that every observable of the irreducible theory, which we found that verifies the equations (52–53) on the surface (38–39), will check these equations also on the surface (54). This means that every observable of the irreducible system is an observable of the theory based on the first-class constraints (54). Conversely, if F represents a physical

observable of the system underlying the constraints (54), then it should check the equations

$$[F, G_{a_0}] \approx 0, \quad [F, \pi_{a_{2k+1}}] \approx 0, \quad k = 0, \dots, a, \quad (55)$$

on the surface (54). Then, it follows that F satisfies the relations

$$[F, \gamma_{a_0}] = [F, G_{a_0}] + [F, A_{a_0}^{a_1}] \pi_{a_1} + [F, \pi_{a_1}] A_{a_0}^{a_1} \approx 0, \quad (56)$$

$$\begin{aligned} [F, \gamma_{a_{2k}}] &= [F, Z_{a_{2k}}^{a_{2k-1}}] \pi_{a_{2k-1}} + [F, \pi_{a_{2k-1}}] Z_{a_{2k}}^{a_{2k-1}} + \\ &[F, A_{a_{2k}}^{a_{2k+1}}] \pi_{a_{2k+1}} + [F, \pi_{a_{2k+1}}] A_{a_{2k}}^{a_{2k+1}} \approx 0, \quad k = 1, \dots, b, \end{aligned} \quad (57)$$

on the same surface. Recalling once again the equivalence between this surface and the one expressed by (38–39), we find that F will verify the equations (56–57) also on the surface (38–39), being therefore an observable of the irreducible system. From the above discussion we conclude that the physical observables of the irreducible theory coincide with those associated with the system subject to the first-class constraints (54). Next, we show that the physical observables of the system possessing the constraints (54) and the ones corresponding to the original reducible theory coincide. In this light, we remark that the surface (54) can be inferred in a trivial manner from (1) by adding the canonical pairs $(y^{a_{2k+1}}, \pi_{a_{2k+1}})_{k=0, \dots, a}$ and requiring that their momenta vanish. Thus, the observables of the original redundant theory are unaffected by the introduction of the new canonical pairs. In fact, the difference between an observable F of the system subject to the constraints (54) and one of the original theory, \bar{F} , is of the type $F - \bar{F} = \sum_{k=0}^a f^{a_{2k+1}} \pi_{a_{2k+1}}$. As any two observables that differ through a combination of first-class constraint functions can be identified, we find that the physical observables of the initial theory coincide with those of the system described by the constraints (54). So far, we proved that the observables of the system with the constraints (54) coincide on the one hand with those of the irreducible theory, and, on the other hand, with those of the original reducible one. In conclusion, the physical observables associated with the irreducible system also coincide with those of the starting on-shell reducible first-class theory. In turn, this result will have a strong impact at the level of the BRST analysis.

In the above we have shown that starting with an arbitrary on-shell reducible first-class Hamiltonian system displaying the standard Hamiltonian

BRST symmetry s_R we can construct a corresponding irreducible first-class theory, whose BRST symmetry s complies with the basic requirements of the Hamiltonian BRST formalism. The previous result on the physical observables induces that the zeroth order cohomological groups of the corresponding BRST symmetries are isomorphic

$$H^0(s_R) \simeq H^0(s). \quad (58)$$

In addition, both symmetries are nilpotent

$$s_R^2 = 0 = s^2. \quad (59)$$

Then, from the point of view of the fundamental equations of the BRST formalism, namely, the nilpotency of the BRST operator and the isomorphism between the zeroth order cohomological group of the BRST differential and the algebra of physical observables, it follows that it is permissible to replace the Hamiltonian BRST symmetry of the original on-shell L -stage reducible system with that of the irreducible theory. Thus, we can substitute the path integral of the reducible system in the Hamiltonian BRST approach by that of the irreducible theory.

However, it would be convenient to infer a covariant path integral with respect to the irreducible system. The present phase-space coordinates may not ensure the covariance. For instance, if we analyze the gauge transformations of the extended action of the irreducible system, we remark that those corresponding to the Lagrange multipliers of the constraint functions γ_{a_0} will not involve the term $-Z_{a_1}^{a_0} \epsilon^{a_1}$, which is present within the reducible context with respect to the constraint functions G_{a_0} . For all known models, the presence of this term is essential in arriving at some covariant gauge transformations at the Lagrangian level. For this reason it is necessary to gain such a term also within the irreducible setting. Moreover, it is possible that some of the newly added canonical variables lack covariant Lagrangian gauge transformations. This signalizes that we need to add more phase-space variables to be constrained in an appropriate manner. In view of this, we introduce the additional bosonic canonical pairs

$$\left(y^{(1)a_{2k+1}}, \pi_{a_{2k+1}}^{(1)}\right), \left(y^{(2)a_{2k+1}}, \pi_{a_{2k+1}}^{(2)}\right), \quad k = 0, \dots, a, \quad (60)$$

$$\left(y^{a_{2k}}, \pi_{a_{2k}}\right), \quad k = 1, \dots, b, \quad (61)$$

subject to some constraints of the type

$$\gamma_{a_{2k+1}}^{(1)} \equiv \pi_{a_{2k+1}} - \pi_{a_{2k+1}}^{(1)} \approx 0, \quad \gamma_{a_{2k+1}}^{(2)} \equiv \pi_{a_{2k+1}}^{(2)} \approx 0, \quad k = 0, \dots, a, \quad (62)$$

$$\gamma_{a_{2k}} \equiv \pi_{a_{2k}} \approx 0, \quad k = 1, \dots, b. \quad (63)$$

In this manner we do not affect in any way the properties of the irreducible theory as (62–63) still form together with (38–39) an irreducible first-class set. The newly added constraints implies the introduction of some supplementary ghosts and antighosts and the extension of the action of the BRST operator on them in the usual manner. Then, there exists a consistent Hamiltonian BRST symmetry with respect to the new irreducible theory, described by the constraints (38–39) and (62–63). Now, if we choose the first-class Hamiltonian with respect to the above first-class constraints in an adequate manner, we can in principle generate a gauge algebra that leads to some covariant Lagrangian gauge transformations. From (44) it results that the former set of constraints in (62) reduces to $\pi_{a_{2k+1}}^{(1)} \approx 0$. Thus, we observe that the surface (38–39), (62–63) results in a trivial way from (38–39) by adding the canonical variables (60–61) and demanding that their momenta vanish. Then, the difference between an observable F of the new irreducible theory and one of the previous irreducible system, \bar{F} , is of the type $F - \bar{F} = \sum_{k=0}^a f^{a_{2k+1}} \pi_{a_{2k+1}}^{(1)} + \sum_{k=0}^a g^{a_{2k+1}} \pi_{a_{2k+1}}^{(2)} + \sum_{k=1}^b h^{a_{2k}} \pi_{a_{2k}}$, hence F and \bar{F} can be identified. Therefore, the physical observables corresponding to the two irreducible systems coincide, such that the supplementary constraints (62–63) do not afflict the previously established equivalence with the physical observables of the original redundant theory. In consequence, we can replace the Hamiltonian BRST symmetry of the original reducible system with that of the latter irreducible theory, and similarly with regard to the associated path integrals.

With all these elements at hand, the Hamiltonian BRST quantization of the irreducible theory goes along the standard lines. Defining a canonical action for s in the usual way as $s \bullet = [\bullet, \Omega]$, with Ω the canonical generator (the BRST charge), the nilpotency of s implies that Ω should satisfy the equation

$$[\Omega, \Omega] = 0. \quad (64)$$

The existence of the solution to the equation (64) is then guaranteed by the acyclicity of the irreducible Koszul-Tate differential at positive antighost

numbers. Once constructed the BRST charge, we pass to the construction of the BRST-invariant Hamiltonian, $H_B = H' + \text{“more”}$, that satisfies $[H_B, \Omega] = 0$, where H' stands for the first-class Hamiltonian with respect to the constraints (38–39) and (62–63). In order to fix the gauge, we have to choose a gauge-fixing fermion K that implements some irreducible gauge conditions. The possibility to construct some irreducible gauge conditions is facilitated by the introduction of the pairs (y, π) , which, at this level, play the same role like the auxiliary variables in the non-minimal approach. Actually, these variables are more relevant than the corresponding non-minimal ones appearing in the gauge-fixing process from the reducible case because they prevent the appearance of the reducibility (via the irreducible first-class constraints), while the non-minimal coordinates in the reducible situation are mainly an effect of the redundancy. The gauge-fixed Hamiltonian $H_K = H_B + [K, \Omega]$ will then produce a correct gauge-fixed action S_K with respect to the irreducible theory. In this way, we showed how a reducible first-class Hamiltonian system can be approach along an irreducible BRST procedure, hence without using either ghosts for ghosts or their antighosts.

Finally, a word of caution. The appearance of the inverse of the matrices $D_{b_k}^{a_k}$ in various formulas implies that our approach may have problems with locality. On the other hand, locality is required for all concrete applications in field theory. However, taking into account the ambiguities present in the choice of the reducibility functions [6] and of the functions $A_{a_k}^{a_{k+1}}$ (see (43)), it might be possible to obtain a local formulation. This completes our treatment.

4 Example

In this section we illustrate the general formalism exposed in the above in the case of a second-stage reducible model. We start with the Lagrangian action

$$S_0^L[A^{\mu\nu\rho}] = \int d^7x \left(-\frac{1}{48} F_{\mu\nu\rho\lambda} F^{\mu\nu\rho\lambda} + \xi \varepsilon_{\mu\nu\rho\lambda\alpha\beta\gamma} F^{\mu\nu\rho\lambda} A^{\alpha\beta\gamma} \right), \quad (65)$$

where

$$F_{\mu\nu\rho\lambda} = \partial_\mu A_{\nu\rho\lambda} - \partial_\nu A_{\mu\rho\lambda} + \partial_\rho A_{\lambda\mu\nu} - \partial_\lambda A_{\rho\mu\nu} \equiv \partial_{[\mu} A_{\nu\rho\lambda]}, \quad (66)$$

$\varepsilon_{\mu\nu\rho\lambda\alpha\beta\gamma}$ represents the completely antisymmetric seven-dimensional symbol and ξ is a constant. From the canonical analysis of action (65) we deduce the first-class constraints

$$\gamma_{ij}^{(1)} \equiv \pi_{0ij} \approx 0, \quad (67)$$

$$G_{ij}^{(2)} \equiv -3 \left(\partial^k \pi_{kij} + \xi \varepsilon_{0ijklmn} F^{klmn} \right) \approx 0, \quad (68)$$

and the first-class Hamiltonian

$$H = \int d^6x \left(\frac{1}{48} F_{ijkl}^2 - 3 \left(\pi_{ijk} - 4\xi \varepsilon_{0ijklmn} A^{lmn} \right)^2 + A^{0ij} G_{ij}^{(2)} \right), \quad (69)$$

where $\pi_{\mu\nu\rho}$ stand for the canonical momenta conjugated with $A^{\mu\nu\rho}$. The notation F_{ijkl}^2 signifies $F_{ijkl} F^{ijkl}$, and similarly for the other square. The constraint functions in (68) are second-stage reducible, the first-, respectively, second-stage reducibility relations being expressed by

$$Z_k^{ij} G_{ij}^{(2)} = 0, \quad Z^k Z_k^{ij} = 0, \quad (70)$$

where the reducibility functions have the form

$$Z_{a_1}^{a_0} \rightarrow Z_k^{ij} = \partial^{[i} \delta_{k]}^{j]}, \quad Z_{a_2}^{a_1} \rightarrow Z^k = \partial^k. \quad (71)$$

Acting like in section 2, we enlarge the original phase-space with the bosonic canonical pairs $(y^{a_1}, \pi_{a_1}) \equiv (H^i, \pi_i)$ and construct an irreducible first-class constraint set corresponding to (68) of the type

$$\gamma_{a_0} \rightarrow \gamma_{ij}^{(2)} \equiv -3 \left(\partial^k \pi_{kij} + \xi \varepsilon_{0ijklmn} F^{klmn} \right) - \partial_{[i} \pi_{j]} \approx 0, \quad (72)$$

$$\gamma_{a_2} \rightarrow \gamma^{(2)} \equiv -\partial^i \pi_i \approx 0. \quad (73)$$

In inferring the above irreducible constraints we took the functions of the type $A_{a_k}^{a_{k+1}}$ like

$$A_{a_0}^{a_1} \rightarrow A_{ij}^k = \partial_{[i} \delta_{j]}^k, \quad A_{a_1}^{a_2} \rightarrow A_k = \partial_k, \quad (74)$$

such that they comply with the requirements from the general theory. As we mentioned in section 3, in order to generate a covariant gauge-fixed action as a result of the Hamiltonian BRST quantization of the irreducible model it is still necessary to enlarge the phase-space with some canonical pairs

subject to additional constraints such that the overall constraint set is first-class and irreducible. In this light, we introduce the canonical pairs (H^0, π_0) , $(H^{(1)i}, \pi_i^{(1)})$, $(H^{(2)i}, \pi_i^{(2)})$ constrained like

$$\gamma^{(1)} \equiv \pi_0 \approx 0, \quad \gamma_i^{(1)} \equiv \pi_i - \pi_i^{(1)} \approx 0, \quad \gamma_i^{(2)} \equiv -2\pi_i^{(2)} \approx 0. \quad (75)$$

Obviously, the constraint set (67), (72–73), (75) is irreducible and first-class. We take the first-class Hamiltonian with respect to this set under the form

$$H' = \int d^6x \left(\frac{1}{48} F_{ijkl}^2 - 3 \left(\pi_{ijk} - 4\xi \varepsilon_{0ijklmn} A^{lmn} \right)^2 + A^{0ij} \gamma_{ij}^{(2)} + H^0 \gamma^{(2)} + H^i \pi_i^{(2)} + H^{(2)i} \left(\partial^j \gamma_{ji}^{(2)} + \partial_i \gamma^{(2)} \right) \right), \quad (76)$$

such that the irreducible gauge algebra will lead to a covariant path integral.

Next, we approach the Hamiltonian BRST quantization of the irreducible model. In view of this, we add the minimal fermionic canonical pairs ghost-antighost $(\eta^{(\Delta)ij}, \mathcal{P}_{ij}^{(\Delta)})$, $(\eta^{(\Delta)i}, \mathcal{P}_i^{(\Delta)})$, $(\eta^{(\Delta)}, \mathcal{P}^{(\Delta)})$, with $\Delta = 1, 2$, associated with the corresponding constraints in (67), (72–73) and (75). All the ghosts possess ghost number one, while their antighosts display ghost number minus one. Moreover, we take a non-minimal sector organized as (P_b^{ij}, b_{ij}) , (P_b, b) , $(P_{b^1}^{ij}, b_{ij}^1)$, (P_{b^1}, b^1) , $(P_{\bar{\eta}}^{ij}, \bar{\eta}_{ij})$, $(P_{\bar{\eta}}, \bar{\eta})$, $(P_{\bar{\eta}^1}^{ij}, \bar{\eta}_{ij}^1)$, $(P_{\bar{\eta}^1}, \bar{\eta}^1)$. The first four sets of non-minimal variables are bosonic with ghost number zero, while the others are fermionic, with the $P_{\bar{\eta}}$'s and $\bar{\eta}$'s of ghost number one, respectively, minus one. Consequently, the non-minimal BRST charge reads as

$$\Omega = \int d^6x \left(\sum_{\Delta=1}^2 \left(\gamma_{ij}^{(\Delta)} \eta^{(\Delta)ij} + \gamma_i^{(\Delta)} \eta^{(\Delta)i} + \gamma^{(\Delta)} \eta^{(\Delta)} \right) + P_{\bar{\eta}}^{ij} b_{ij} + P_{\bar{\eta}} b + P_{\bar{\eta}^1}^{ij} b_{ij}^1 + P_{\bar{\eta}^1} b^1 \right). \quad (77)$$

The BRST-invariant Hamiltonian corresponding to (76) is given by

$$H_B = \int d^6x \left(\eta^{(1)ij} \mathcal{P}_{ij}^{(2)} + \eta^{(1)} \mathcal{P}^{(2)} - \frac{1}{2} \eta^{(1)i} \mathcal{P}_i^{(2)} + \frac{1}{2} \eta^{(2)ij} \partial_{[i} \mathcal{P}_{j]}^{(2)} + \frac{1}{2} \eta^{(2)} \partial^i \mathcal{P}_i^{(2)} - 2\eta^{(2)i} \left(\partial^j \mathcal{P}_{ji}^{(2)} + \partial_i \mathcal{P}^{(2)} \right) \right) + H'. \quad (78)$$

In order to determine the gauge-fixed action, we work with the gauge-fixing fermion

$$K = \int d^6x \left(\mathcal{P}_{ij}^{(1)} \left(\partial_k A^{kij} + \frac{1}{2} \partial^{[i} H^{(1)j]} \right) + \mathcal{P}^{(1)} \partial_i H^{(1)i} + \right.$$

$$\begin{aligned} & \mathcal{P}_i^{(1)} \left(2\partial_j A^{j0} + \partial^i H^0 \right) + P_{b^1}^{ij} \left(\mathcal{P}_{ij}^{(1)} - \bar{\eta}_{ij} + \dot{\bar{\eta}}_{ij}^1 \right) + \\ & P_{b^1} \left(\mathcal{P}^{(1)} - \bar{\eta} + \dot{\bar{\eta}}^1 \right) + P_b^{ij} \left(\bar{\eta}_{ij}^1 + \dot{\bar{\eta}}_{ij} \right) + P_b \left(\bar{\eta}^1 + \dot{\bar{\eta}} \right). \end{aligned} \quad (79)$$

It is clear that the above gauge-fixing fermion implements some irreducible canonical gauge conditions. Introducing the expression of the gauge-fixed Hamiltonian $H_K = H_B + [K, \Omega]$ in the gauge-fixed action and eliminating some auxiliary variables on their equations of motion, we finally find the path integral

$$Z_K = \int \mathcal{D}A^{\mu\nu\rho} \mathcal{D}H^{(1)\mu} \mathcal{D}b_{\mu\nu} \mathcal{D}b \mathcal{D}\eta^{(2)\mu\nu} \mathcal{D}\eta^{(2)} \mathcal{D}\bar{\eta}_{\mu\nu} \mathcal{D}\bar{\eta} \exp(iS_K), \quad (80)$$

where

$$\begin{aligned} S_K = & S_0^L[A^{\mu\nu\rho}] + \int d^7x \left(b_{\mu\nu} \left(\partial_\rho A^{\rho\mu\nu} + \frac{1}{2} \partial^{[\mu} H^{(1)\nu]} \right) + \right. \\ & \left. b \partial_\mu H^{(1)\mu} - \bar{\eta}_{\mu\nu} \square \eta^{(2)\mu\nu} - \bar{\eta} \square \eta^{(2)} \right). \end{aligned} \quad (81)$$

In deriving (81) we performed the identifications

$$H^{(1)\mu} = (H^0, H^{(1)i}), \quad b_{\mu\nu} = (\pi_i^{(1)}, b_{ij}), \quad (82)$$

$$\bar{\eta}_{\mu\nu} = (-\mathcal{P}_i^{(1)}, \bar{\eta}_{ij}), \quad \eta^{(2)\mu\nu} = (\eta^{(2)i}, \eta^{(2)ij}), \quad (83)$$

and used the symbol $\square = \partial_\rho \partial^\rho$. It is clear that the gauge-fixed action (81) is covariant and displays no residual gauge invariances although we have not used any ghosts for ghosts. It is precisely the introduction of the supplementary canonical pairs constrained accordingly to (75) and the choice (76) of the first-class Hamiltonian that generates a Hamiltonian gauge algebra implementing the covariance. Actually, the gauge-fixed action (81) can be alternatively inferred in the framework of the antifield BRST treatment if we start with the action

$$S_0^L[A^{\mu\nu\rho}, H^{(1)\mu}] = S_0^L[A^{\mu\nu\rho}], \quad (84)$$

subject to the irreducible gauge transformations

$$\delta_\epsilon A^{\mu\nu\rho} = \partial^{[\mu} \epsilon^{\nu\rho]}, \quad \delta_\epsilon H^{(1)\mu} = 2\partial_\nu \epsilon^{\nu\mu} + \partial^\mu \epsilon, \quad (85)$$

and employ an adequate non-minimal sector and gauge-fixing fermion. In fact, the irreducible and covariant gauge transformations (85) are the Lagrangian result of our irreducible Hamiltonian procedure inferred via the extended and total actions and their gauge transformations.

In the end, let us briefly compare the above results with those derived in the standard reducible BRST approach. The gauge-fixed action in the usual reducible BRST framework can be brought to the form

$$S_\psi = S_0^L [A^{\mu\nu\rho}] + \int d^7x \left(\mathcal{B}_{\mu\nu} \left(\partial_\rho A^{\rho\mu\nu} + \frac{1}{2} \partial^{[\mu} \varphi^{\nu]} \right) + \mathcal{B} \partial_\mu \varphi^\mu - \bar{C}_{\mu\nu} \square C^{\mu\nu} - \bar{C}^2 \square \bar{C}^1 - \bar{C}_\mu \square C^\mu - \bar{C} \square C \right), \quad (86)$$

where $C^{\mu\nu}$ stand for the ghost number one ghosts, C^μ represent the ghost number two ghosts for ghosts, and C is the ghost number three ghost for ghost for ghost. The remaining variables belong to the non-minimal sector and have the properties

$$\epsilon(\mathcal{B}_{\mu\nu}) = 0, \quad gh(\mathcal{B}_{\mu\nu}) = 0, \quad \epsilon(\varphi^\mu) = 0, \quad gh(\varphi^\mu) = 0, \quad (87)$$

$$\epsilon(\mathcal{B}) = 0, \quad gh(\mathcal{B}) = 0, \quad \epsilon(\bar{C}_{\mu\nu}) = 1, \quad gh(\bar{C}_{\mu\nu}) = -1, \quad (88)$$

$$\epsilon(\bar{C}_\mu) = 0, \quad gh(\bar{C}_\mu) = -2, \quad \epsilon(\bar{C}) = 1, \quad gh(\bar{C}) = -3, \quad (89)$$

$$\epsilon(\bar{C}^2) = 1, \quad gh(\bar{C}^2) = -1, \quad \epsilon(\bar{C}^1) = 1, \quad gh(\bar{C}^1) = 1. \quad (90)$$

By realizing the identifications

$$C^{\mu\nu} \leftrightarrow \eta^{(2)\mu\nu}, \quad \mathcal{B}_{\mu\nu} \leftrightarrow b_{\mu\nu}, \quad \varphi^\mu \leftrightarrow H^{(1)\mu}, \quad (91)$$

$$\mathcal{B} \leftrightarrow b, \quad \bar{C}_{\mu\nu} \leftrightarrow \bar{\eta}_{\mu\nu}, \quad \bar{C}^2 \leftrightarrow \bar{\eta}, \quad \bar{C}^1 \leftrightarrow \eta^{(2)}, \quad (92)$$

among the variables involved with the gauge-fixed actions inferred within the irreducible and reducible approaches, (81), respectively, (86), the difference between the two gauge-fixed actions is given by

$$S_K - S_\psi = \int d^7x \left(\bar{C}_\mu \square C^\mu + \bar{C} \square C \right). \quad (93)$$

We remark that $S_K - S_\psi$ is proportional to the ghosts for ghosts C^μ and the ghost for ghost for ghost C , which are some essential compounds of the reducible BRST quantization. Although identified at the level of the

gauge-fixed actions, the fields φ^μ and $H^{(1)\mu}$ play different roles within the two formalisms. More precisely, the presence of $H^{(1)\mu}$ within the irreducible model prevents the appearance of the reducibility, while φ^μ is nothing but one of its effects. In fact, the effect of introducing the fields $H^{(1)\mu}$ is that of forbidding the appearance of zero modes which exist within the original reducible theory by means of the first term present in the gauge transformations of $H^{(1)\mu}$ from (85). Indeed, if we take $\epsilon^{\mu\nu} = \partial^{[\mu} \epsilon^{\nu]}$ in (85), then $\partial_\mu \epsilon^{\mu\nu}$ is non-vanishing, such that the entire set of gauge transformations is irreducible. In consequence, all the quantities linked with zero modes, like the ghosts with ghost number greater than one or the pyramid-like structured non-minimal sector, are discarded when passing to the irreducible setting. This completes the analysis of the investigated model.

5 Conclusion

In conclusion, in this paper we have shown how systems with reducible first-class constraints can be quantized by using an irreducible Hamiltonian BRST formalism. The key point of our approach is given by the construction of a Hamiltonian Koszul-Tate complex that emphasizes an irreducible set of first-class constraints. As the physical observables associated with the irreducible and reducible versions coincide, the main equations underlining the Hamiltonian BRST formalism make legitimate the substitution of the BRST quantization of the reducible theory by that of the irreducible one. The canonical generator of the irreducible BRST symmetry exists due to the acyclicity of the irreducible Koszul-Tate differential, while the gauge-fixing procedure is facilitated by the enlargement of the phase-space with the canonical pairs of the type (y, π) . The general formalism is exemplified on a second-stage reducible model involving three-form gauge fields, which is then compared with the results from the standard reducible BRST approach.

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